Publisher's Response to EdReports Review of *HMH Math Expressions* ©2018 (Grades 3–6)

Houghton Mifflin Harcourt

Introduction

Math Expressions ©2018 is based on extensive research funded by the National Science Foundation. It is a proven curriculum designed to improve student learning and engagement and we believe the final published report of "Meets Expectations" for Grades 3–5 provides a resounding endorsement of *Math Expressions.*

Dr. Karen Fuson and the entire team at Houghton Mifflin Harcourt are grateful for the in-depth review performed by the EdReports committee and we appreciate the opportunity to highlight instances, particularly in Grade 6, where we believe the program did in fact meet the stated criteria of the Scoring Guide.

Indicator 1c

Grade 3

EdReports: The instructional materials … partially meet expectations that supporting work enhances focus and coherence simultaneously by engaging students in the major work of the grade.

EdReports cites three examples of missed opportunities to connect multiplication and division (3.OA.A) to one- and two-step word problems using scaled bar graphs and scaled pictographs (3.MD.3).

1. *Unit 1, Lesson 19 missed the opportunity to multiply and divide to solve one- and two-step word problems (3.OA.A) with the scaled pictograph.* However, 3.MD.3 only says that word problems should be solved for scaled bar graphs, not for scaled pictographs.
2. *In Unit 4, Lesson 15, Problem 5 students use multiplication to solve the first step but that work is not directly related to the scaled bar graph.* However, the information in the bar graph as well as the word problem is used to find the solution.
3. *Students create a bar graph for the Unit 4 Performance Assessment (page 577). This is identified as a missed opportunity for multiplication and division word problems.* However, the focus in this Performance Task was on choosing the scale and making the bar graph and on other kinds of graphs done in this task. Furthermore, related word problems were purposefully avoided because possible errors in a student's bar graph could introduce errors in the problems.

Grade 4

EdReports: The instructional materials … partially meet expectations that supporting work enhances focus and coherence simultaneously by engaging students in the major work of the grade.

While examples are given in the review for how *Math Expressions* does meet Indicator 1c, there are three missed opportunities cited. The first two examples are of not giving opportunities to add and subtract fractions. These examples both come from Unit 5. But students do not learn to add and subtract fractions until the following unit, Unit 6, so these are not missed opportunities.

In the third example from Unit 7, Lesson 4, Problem 2, the reviewers suggested that students identify the factor pairs of 8 and 12 (4.OA.4) in order to determine if multiplying the numerator and denominator by 4 leads to a viable solution (4.NF.A). But Problem 2 is related to Problem 1 and is a written recording of the visual method in Problem 1. So finding all factor pairs is unnecessary and would sideline the conceptual focus of the relationship between these two problems.
There are many lessons throughout *Math Expressions* where Indicator 1c is met, including those identified by the reviewers.

Indicator 2f

EdReports: The materials do not attend to the full meaning of Mathematical Practice 5.

The EdReports Evidence Guides state the following: "Every instance of a Mathematical Practice being marked does not necessarily have to encompass the full meaning of a Mathematical Practice, but taken together there should be evidence that the materials carefully attend to the full meaning of each practice."

For Mathematical Practice 5, *Math Expressions* gives students repeated experiences as each tool is introduced, building a foundation for choosing tools strategically later. Then students are encouraged to choose tools that make the most sense to them and to explain the reasoning behind their choices. These are tools as described in Mathematical Practice 5, and they are used frequently throughout the program and chosen by students.

Here are some examples of instructional features that support student choice of tools:

Math Talk in Action models for teachers how to encourage students to think about different tools. **Math Talk** features and **Teacher Notes** encourage teachers to remind students they can choose the tool to use.

Teacher Edition pages

- **Grade 3**
  183, 300, 611–612, 658
- **Grade 4**
  52, 395, 579, 634
- **Grade 5**
  page 77 (Anytime Problem), 120
- **Grade 6**
  130

Student Activity Book pages

- **Grade 5**
  50
- **Grade 6**
  117, 130, 131, 149, 290, 591, 726

In-Depth Inquiry-Based Task Cards

- **Grade 3**
  Unit 4
- **Grade 4**
  Unit 6
- **Grade 6**
  Unit 6;

Homework and Remembering pages

- **Grade 6**

Lessons 3.5, 3.9, 5.16, 7.6, 7.10, 7.11

Math Activity Center

- **Grade 5**

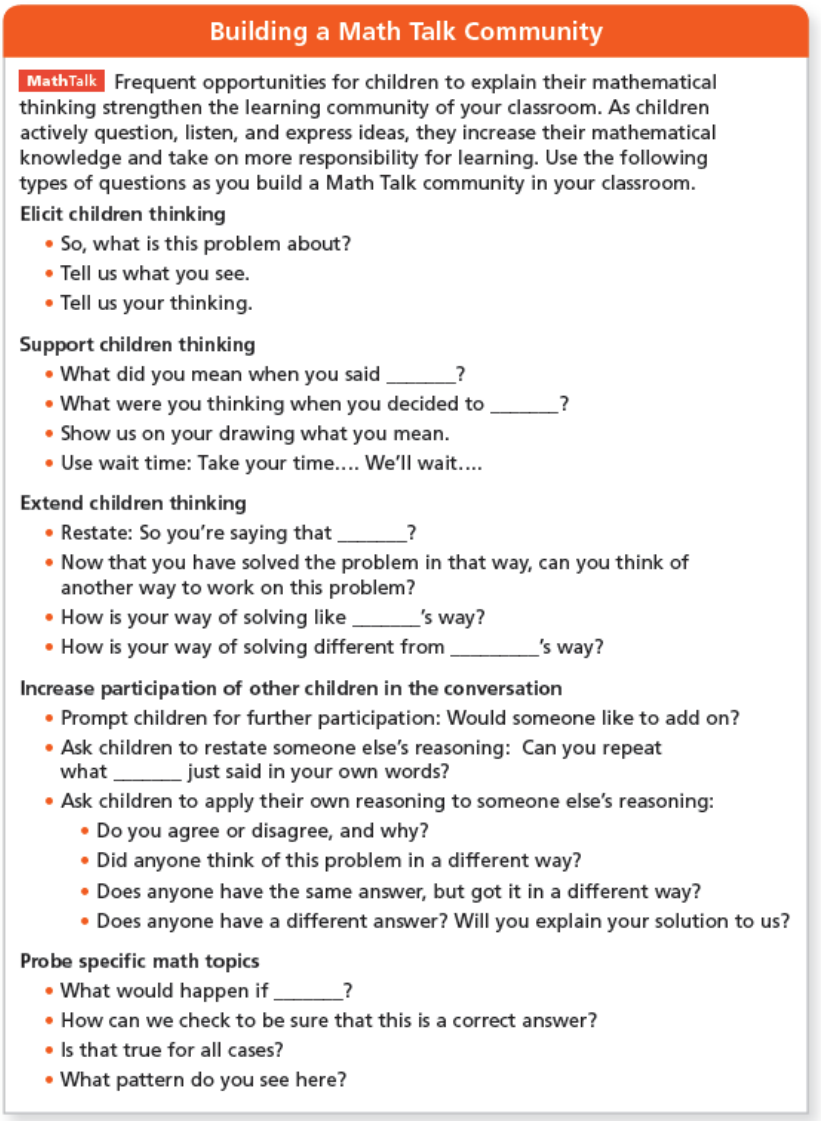
Lesson 1.3 (Writing Prompt)

Indicator 2g.ii

EdReports: The instructional materials …partially meet expectations that the instructional materials assist teachers in engaging students to construct viable arguments and analyze the arguments of others concerning key grade-level mathematics.

The EdReports Evidence Guide indicates that evidence includes teacher prompts and suggested questions, stating that "The materials might guide teachers to ask students to explain their thinking or justify their solutions" and "might prompt teachers to have students look at a solution and decide if it is correct or incorrect and explain why."As EdReports states, In *Math Expressions,* "Math Talk is a critical component of the instructional materials and presents opportunities for students to use a consistent structure: Solve, Explain, Question, and Justify."

The perfect scores attained for Gateway 3 also indicate that *Math Expressions* enhances a teacher's ability to promote student learning. Student explanations are followed by questions from peers and further justification. Instructions for setting up a **Math Talk Community** are shown below. Teachers use these questions all year although assistance is given frequently provided at other points.



Building a Math Talk Community

MathTalk Frequent opportunities for children to explain their mathematical thinking strengthen the learning community of your classroom. As children actively question, listen, and express ideas, they increase their mathematical knowledge and take on more responsibility for learning. Use the following types of questions as you build a Math Talk community in your classroom.

Elicit children thinking

- So, what is this problem about?
- Tell us what you see.
- Tell us your thinking.

Support children thinking

- What did you mean when you said _____?
- What were you thinking when you decided to _____?
- Show us on your drawing what you mean.
- Use wait time: Take your time.... We'll wait....

Extend children thinking

- Restate: So you're saying that _____?
- Now that you have solved the problem in that way, can you think of another way to work on this problem?
- How is your way of solving like _____'s way?
- How is your way of solving different from _____'s way?

Increase participation of other children in the conversation

- Prompt children for further participation: Would someone like to add on?
- Ask children to restate someone else's reasoning: Can you repeat what _____ just said in your own words?
- Ask children to apply their own reasoning to someone else's reasoning:
  - Do you agree or disagree, and why?
  - Did anyone think of this problem in a different way?
  - Does anyone have the same answer, but got it in a different way?
  - Does anyone have a different answer? Will you explain your solution to us?

Probe specific math topics

- What would happen if _____?
- How can we check to be sure that this is a correct answer?
- Is that true for all cases?
- What pattern do you see here?

The extensive **Unit Overviews** describe visual and conceptual support to assist teachers in supporting students in explaining/justifying and questioning/critiquing. **Math Talk in Action** models student discussion to support developing students' ability to construct arguments and analyze responses. There is frequent support for teachers in this feature, and each Big Idea section of a unit includes an example. **Teacher Notes** provide support for discussions about student thinking or leading **Math Talk**. Some examples are listed below:

- **Grade 3**
  Teacher Edition: pages 116, 312, 319, 663, 680, 722
- **Grade 4**
  Teacher Edition: pages 73, 79, 106, 108, 178, 415
- **Grade 5**
  Teacher Edition: pages 59, 134, 252, 424, 535, 562
- **Grade 6**
  Teacher Edition: pages 204, 277, 316, 510, 642

Indicator 2g.iii

Grade 6
EdReports: The instructional materials … partially meet expectations that materials use accurate mathematical terminology.

Two mathematical terms were considered inaccurate by the EdReports review: *basic ratio* and *cross-multiplication*. Here is the reasoning and research behind HMH's inclusion of these terms in *Math Expressions*.

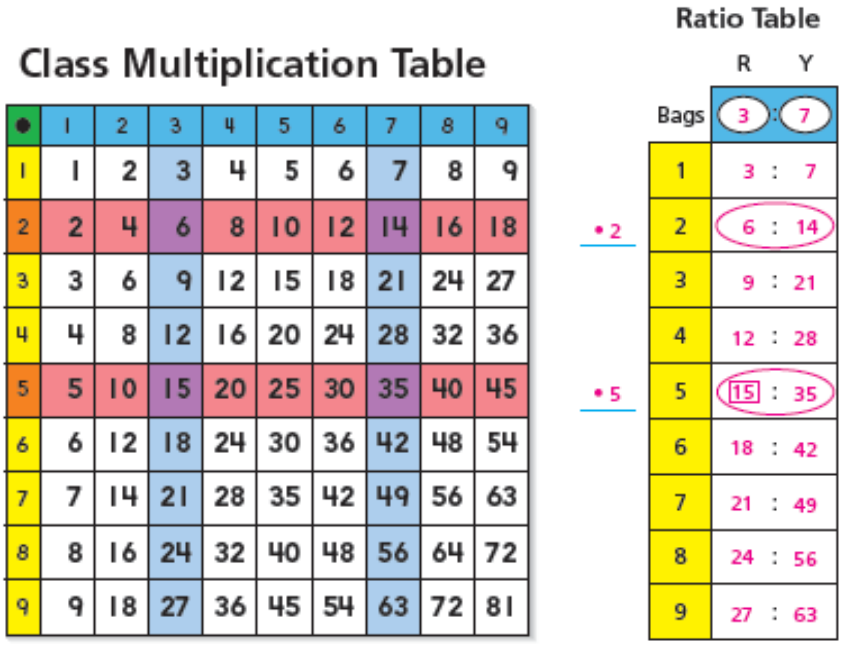
Basic Ratio

Grade 6 carefully develops concepts of rate, ratio, and proportion and relates them to fractions. In explaining how equivalent ratios shown in ratio tables can be found, a term is needed to refer to the ratio pair made from the smallest whole numbers. Jere Confrey's research shows students used the term "the littlest recipe" for such ratio pairs, but we needed a more general term and could find none used frequently, let alone universally.[1]

Standard 6.RP.A.3a requires that students make tables of equivalent ratios relating quantities with whole-number measurements. To address the standard, *Math Expressions* uses a class multiplication table with red and blue strips highlighting rows and columns of the multiplication table to relate to highlighted rows in a ratio table. A term for the smallest whole number ratio is needed because it occurs in the top row of the multiplication table, thus showing how a ratio table relates to a multiplication table.

[1] Confrey, Jere; Scarano, Grace Hotchkiss. *Splitting Reexamined: Results from a Three-Year Longitudinal Study of Children in Grades Three to Five.* Paper Presented at the Annual Meeting of the North American Chapter of the International Group of the Psychology of Mathematics Education (17th, Columbus, OH, October 21–24, 1995.)

Unit 1, Lesson 10



Class Multiplication Table

•	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Ratio Table

	Bags	R : Y
	1	3 : 7
• 2	2	6 : 14
	3	9 : 21
	4	12 : 28
• 5	5	15 : 35
	6	18 : 42
	7	21 : 49
	8	24 : 56
	9	27 : 63

In the classroom research conducted for *Math Expressions,* the term *basic ratio* was used by students to describe this smallest whole number. The term allows students to understand and explain that they can make a table of equivalent ratios by multiplying the basic ratio row. We used *basic ratio* because students used it. It is a meaningful and useful mathematical term that attends to the specialized language and concepts of mathematics. This term has been used in presentations and discussions with mathematicians and mathematics educators (e.g., at the Annual Conference of the National Council of Teachers of Mathematics, 2012) without any objections.

Unit 7 Lesson 1 shows how the term *basic ratio* is used to explain and find equivalent ratios. Teachers found the general term *basic ratio* used over a wide range of ratio and proportion situations to be understandable and useful.

ACTIVITY 2 20m

Use Tables to Compare Ratios

Common Core State Standards
Mathematical Content
6.RP.A.1, 6.RP.A.3, 6.RP.A.3.a
Mathematical Practices
MP4, MP5, MP7, MP8

Goal
Compare ratios using tables of equivalent ratios.

Materials
Student Activity Book p. 337, MathBoard materials

Review Tables of Equivalent Ratios

Begin by explaining the scenario that will be used in this lesson.

- Sue's favorite cherry-orange drink recipe has the ratio 5 cups cherry juice to 4 cups orange juice.
- Ben's favorite cherry-orange drink recipe has the ratio 6 cups cherry juice to 5 cups orange juice.

MP4, MP5 **Model Mathematics/Use Appropriate Tools | MathBoard** Copy the ratio tables in the next column on the board while students copy them on their MathBoards. Students work together in groups to complete the tables so that each row in the ratio table for Sue's drink will show a cherry-orange ratio that is equivalent to the basic ratio 5:4. Each row in the ratio table for Ben's drink will show a cherry-orange ratio that is equivalent to the ratio 6:5.

In this unit, we will be transitioning from vertical tables with two columns to horizontal tables with two rows and from colon notation to fraction notation for ratios. For this reason, we will not be using the colons along the vertical rule of the table.

Go over the answers. If there is disagreement over some of the answers, ask students who disagree to show their solutions and discuss as a class why some solutions are not correct.

Sue's Drink

Cherry	Orange
5	4
10	8
25	20
45	36
15	12
30	24
40	32

Ben's Drink

Cherry	Orange
6	5
48	40
30	25
12	10
24	20
60	50
18	15

Discuss the Meaning of Equivalent Ratios

MP7 **Look for Structure | Identify Relationships** Emphasize the key point that Sue's mixture has the same cherry-orange flavor for every row of her ratio table and, in fact, for every possible row. This is what is meant by equivalent ratios.

Students should understand that if Sue is making her special drink using the basic ratio, she mixes 5 cups of cherry and 4 cups of orange. The next row in the ratio table, the 10:8 row, represents 10 cups of cherry juice mixed with 8 cups of orange juice. To get this row, we are doubling the basic recipe. That is, we are making two batches. Each batch has the same cherry-orange flavor, so the whole mixture has the same cherry-orange flavor.

The same applies to Ben's mixture. Each row of his ratio table represents a drink that tastes the same as the drink he makes using the basic ratio.

Cross-multiplication

There have been discussions in mathematics education for decades about the use of the term *cross-multiplication*, which is rarely explained or taught meaningfully. *Cross-multiplication* is discussed because we found during our research that many families used this term and taught this method at home to their children by rote, because they had never had the opportunity to make conceptual sense of the method. At all grades, *Math Expressions* introduces methods and terms that come from students' homes but provides meaningful opportunities to build conceptual understanding by relating these to other methods students have already learned and discussed.

Through the instructional progression in *Math Expressions,* cross-multiplying is not a mysterious method to students, but a writing of the product of products made from the same four factors from the rows and columns of that proportion within the Multiplication Table. Students use multiplication tables with strips to make proportions.

Unit 7 Lesson 3
Teacher Edition, page 703

Cups of Trail Mix
Walnuts : Raisins

3	5
	1
6	10
21	

	3	5
2	6	10
7	21	35

Class Multiplication Table

•	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
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6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

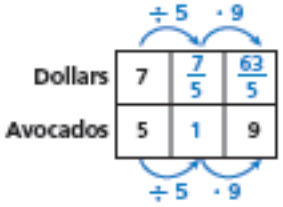
Students also relate the cross-multiplying method to other methods they have used earlier to represent and solve proportions so that the cross-multiplication method becomes even more understandable. The Factor Puzzle shows its factors written outside the rows and columns, so this method is a reminder that the products from within the table used in cross-multiplication come from the same four factors written in a different order.

Unit 7 Lesson 4
Student Activity Book, page 346

9 Zander paid $7 for 5 avocados. How much would 9 avocados cost?

Discuss how these solution strategies relate to each other.

Rate Table



Factor Puzzle

Cross-Multiplication

$c \cdot 5 = 7 \cdot 9$

$5c = 63$

$c = \frac{63}{5}$

The price for 9 avocados is $\frac{63}{5}$ dollars, or $12.60.

Thus, the treatment of cross-multiplication explains why cross-multiplication works. The **Teacher Note** below emphasizes that cross-multiplication is to be understood conceptually with mathematical reasoning and is not taught as a rote method.

Unit 7 Lesson 4
Teacher Edition, page 711

What to Expect from Students It is important that students understand cross-multiplication and do not just use it as a rote method. Connect cross-multiplication to other methods throughout the unit and continue to elicit from students that cross-multiplication works because the products on each side of the equation are just the products of the four factors that make up the Factor Puzzle for the proportion.

We think programs must discuss mathematical terms that students bring from home and help students understand these terms and relate them to other mathematical vocabulary. We also think it is appropriate to include terms that students invent if there is a need for such terms and there is no standard vocabulary for that term.